Rotational Model for Oscillating Waves Over the Surface of A Sphere

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Introduction

- Wave oscillations in coupled networks find themselves in different biological environments
- Ex: Brain Wave Activity, Calcium deposits on frog oocyte surfaces
- Oscillations can be captured on electroencephalograms (EEGs)
Wave Rotation in EEGs

\[ \alpha-\text{EEG: rotating waves} \]

0.55  0.32  0.09  0.02  0.01
Mathematical Background

- Weakly-coupled limit cycle oscillators reduced to scalar equations
- Must Prove to analyze data
Research Goals

- Oscillators tend toward synchronous movement
- To create a model that has non-synchronous solutions
- To determine how often these non-synchronous solutions occur in differing models (n=20 and n=64 oscillators)
Dodecahedron: A Simple Model

- Is a good model to start with:
  - 20 points evenly distributed on surface
  - Exactly 3 neighbors for each of the vertices
Dodecahedron: A Simple Model

- $x_{1p} = c + H(x_{8} - x_{1}) + H(x_{5} - x_{1}) + H(x_{2} - x_{1})$
- $x_{1'} = -x_{1}$
- $x_{2'} = c + H(x_{10} - x_{2}) + H(x_{3} - x_{2}) + H(x_{1} - x_{2}) - x_{1p}$
- $x_{3'} = c + H(x_{12} - x_{3}) + H(x_{4} - x_{3}) + H(x_{2} - x_{3}) - x_{1p}$
- $x_{4'} = c + H(x_{14} - x_{4}) + H(x_{5} - x_{4}) + H(x_{3} - x_{4}) - x_{1p}$
- $x_{5'} = c + H(x_{6} - x_{5}) + H(x_{4} - x_{5}) + H(x_{1} - x_{5}) - x_{1p}$
- $x_{6'} = c + H(x_{15} - x_{6}) + H(x_{7} - x_{6}) + H(x_{5} - x_{6}) - x_{1p}$
- $x_{7'} = c + H(x_{17} - x_{7}) + H(x_{8} - x_{7}) + H(x_{6} - x_{7}) - x_{1p}$
- $x_{8'} = c + H(x_{9} - x_{8}) + H(x_{7} - x_{8}) + H(x_{1} - x_{8}) - x_{1p}$
- $x_{9'} = c + H(x_{18} - x_{9}) + H(x_{10} - x_{9}) + H(x_{8} - x_{9}) - x_{1p}$
- $x_{10'} = c + H(x_{11} - x_{10}) + H(x_{9} - x_{10}) + H(x_{2} - x_{10}) - x_{1p}$
- $x_{11'} = c + H(x_{19} - x_{11}) + H(x_{12} - x_{11}) + H(x_{10} - x_{11}) - x_{1p}$
- $x_{12'} = c + H(x_{13} - x_{12}) + H(x_{11} - x_{12}) + H(x_{3} - x_{12}) - x_{1p}$
- $x_{13'} = c + H(x_{20} - x_{13}) + H(x_{14} - x_{13}) + H(x_{12} - x_{13}) - x_{1p}$
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- $x_{15'} = c + H(x_{16} - x_{15}) + H(x_{14} - x_{15}) + H(x_{6} - x_{15}) - x_{1p}$
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- $x_{19'} = c + H(x_{20} - x_{19}) + H(x_{18} - x_{19}) + H(x_{11} - x_{19}) - x_{1p}$
- $x_{20'} = c + H(x_{19} - x_{20}) + H(x_{16} - x_{20}) + H(x_{13} - x_{20}) - x_{1p}$

$H(x) = \sin(x)$
Theorem (from an earlier Ermentrout paper):

Let $H(x)$ be an odd periodic function:

- $H(x+2\pi) = H(x)$
- $H(-x) = -H(x)$

If these functions are weakly coupled in a network, then there exists a real world solution to this function that is asymptotically stable.

So if we use this model, we know there exist non-synchronized asymptotically stable solutions.
Platonic Solids
N=64 Model

- Difficult to evenly distribute 64 points about a sphere
- Neil Sloane-algorithm for finding coordinates of evenly distributed points on sphere’s surface
- C program created to find a neighborhood of fixed radius around each point
- Generated connection matrix (5-7 neighbors)
Results

- Our goal was to find out if non-synchronous solutions exist, and if so, how often they occur.
- For the Dodecahedron Model, ~2% of the time non-synchronous solutions occur.
- For the n=64 oscillator model, ~10% of the time non-synchronous solutions occur.
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Synchronous vs. Non-synchronous Movement
Conclusions

- We found a successful model that produced non-synchronous solutions.
- We have an idea of how often these solutions occur for two different models.
Future Work

Future research on this topic can focus on:

- Observing higher numbers of n oscillators on the sphere’s surface
- Utilizing different periodic functions other than \( \sin(x) \)
- Placing not only oscillators but excitable or other active elements
- Altering the connectivity of networks to see if this affects the probability of synchronization
- Addition of long range connections with delays to see its effect (Connections common in the brain)
References

Thank you.

Any Questions?