Beyond a pacemaker’s entrainment limit: phase walk-through

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**Fireflies**

“A great belt of light, some ten feet wide, formed by thousands upon thousands of fireflies whose green phosphorescence bridges the shoulder-high grass. . . The fluorescent band composed of these tiny organisms lights up and goes out with a precision that is perfectly synchronized.”

- Joy Adamson
Examples of Biological Synchronization

- Pacemaker cells in the heart
- Discharging of brain cells during epileptic seizures
- Women’s menstrual cycles
- Hair growth in rodents
Objectives

- Mathematically model the entrainment, loss of entrainment, and phase walk-through for coupled pace-makers
- Determine the relationship between frequency of the zeitgeber and the response of the oscillator
On the small scale

- $T_0 =$ natural period of oscillator
- $T =$ period of stimulus
- $D\theta/dt = 2\pi/T_0$
Entrainment and loss of entrainment

- Depends on the difference between the natural frequency of the oscillator and the frequency of the zeitgeber
  - $T > T_e$ – synchronization
  - $T < T_e$ - desynchronization
- Can synchronize out of phase
Phase walk-through

- The phase difference between the oscillator and the stimulus varies periodically
Coupled frequency

\[
\frac{d\theta}{dt} = \frac{2\pi}{T_0} + \beta \sin(\omega t - \theta)
\]

\[\phi = \omega t - \theta\]

\[\omega = \frac{2\pi}{T}\]
On the large scale

\[ \frac{d\theta}{dt} = \omega - \frac{d\phi}{dt} \]

\[ \frac{d\phi}{dt} = \frac{2\pi}{T_0 T} (T_0 - T) - \beta \sin(\phi) \]
Change in phase difference

$T_0 - T$ small
Change in Phase Difference Cont

$T_0 - T$ large
Nested oscillations

Beats

\[ T_b = \frac{C(T_e, T_0)}{\sqrt{T_e} \left( \frac{T_e}{T} \right) - 1} \]
Important Notes

• Just because $\phi$ is periodic, $\theta$ does not necessarily have a period of $T_{\text{beat}}$
• Results apply to weakly driven oscillators
• Phase walk through can occur if $T$ is slightly above or slightly below $T_e$. 
Conclusions

- Fireflies serve as a convenient model for coupled oscillators
- $T > T_e$, synchronization
- $T - T_e$ small, phase walk-through
- $T < T_e$, desynchronization
Possible Applications

- Intestine – frequency plateaus
- Circadian rhythms
- Psychology
  - Bipolar disorder
  - SAD
References


References